Open questions for chromatic PSFs

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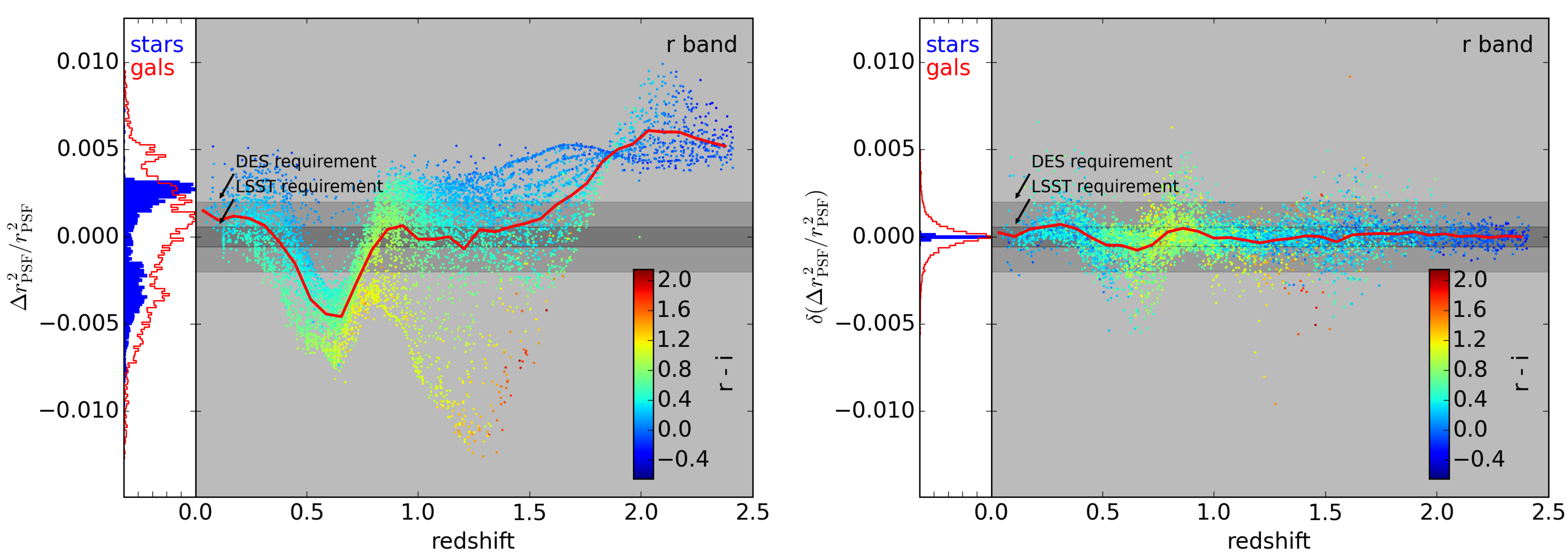
- Precision measurements of galaxy shapes depend on measurements of stars since galaxy images are deconvolved with stellar-estimated PSF.
- Approach assumes stellar PSF is same as galaxy PSF, which is untrue if PSF depends on wavelength.
- Can correct provided:
  - Exact wavelength dependence of PSF
  - SED of each star and galaxy.
- Can use photometry to estimate corrections using a machine learning algorithm trained with a realistic catalog of SEDs and known PSF wavelength dependence.



Impact of chromatic seeing on cosmic shear. **Left**: relative size of PSF for different SEDs at different redshifts. Requirements for LSST and DES are shown as bands. The running mean in red must fall in this band for this single systematic effect to not exceed the statistical uncertainty of LSST cosmic shear. **Right**: residual relative PSF size after employing a machine learning correction using photometry as input.

Open chromatic questions

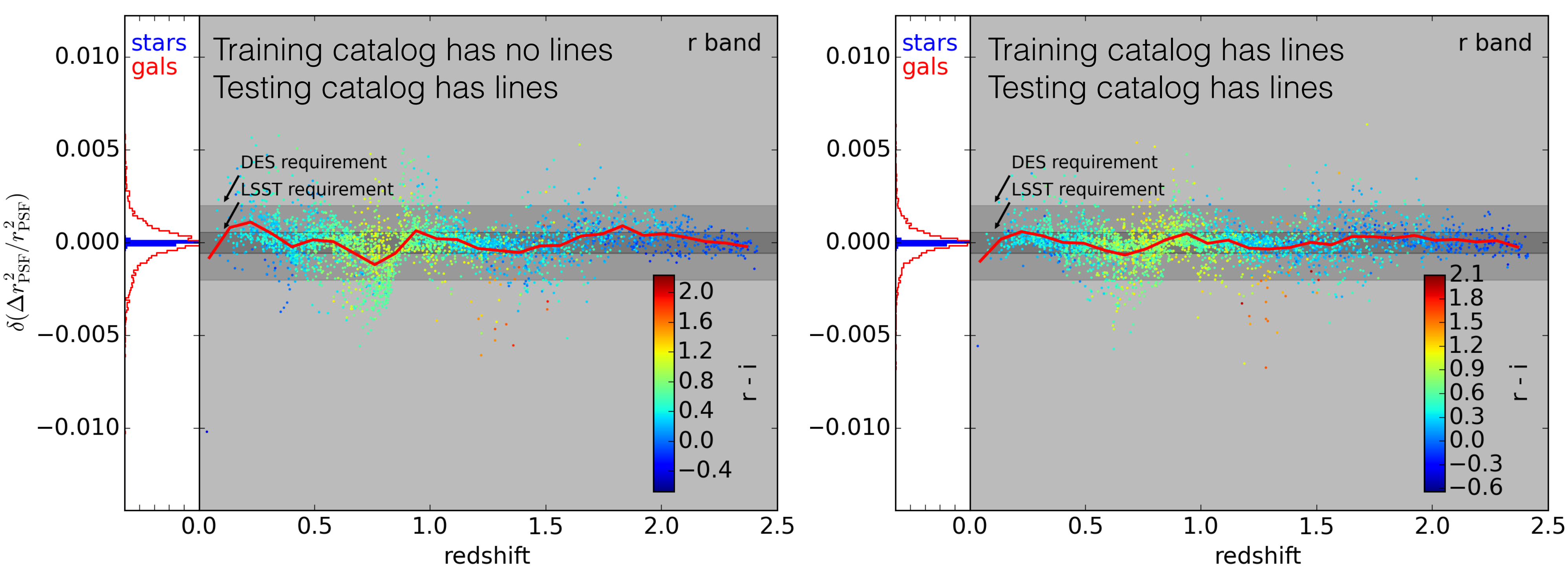
- How does chromatic seeing change with the turbulence outer scale? How isotropic is chromatic seeing?
- LSST optics will exhibit wavelength-dependent refraction, diffraction, and aberrations. How large are these effects?
- Do we need to know PSF wavelength-dependence *a priori?* How can we learn PSF chromaticity from data directly?
- How do zeropoint uncertainties or emission lines in SEDs affect machine learning corrections?
- How do chromatic PSFs combined with galaxies with color gradients affect cosmic shear? (See Sowmya Kamath's poster)
- How can we study realistic (non-parametric) galaxies with color gradients?

Emission lines

- SEDs used by Meyers&Burchat(2015) (from LSST CatSim), do not include emission lines.

- Chromatic PSF biases depend on wavelength of emission line within filter, but photometry does not. Therefore, emission lines may complicate photometry-based machine learning corrections.

- Below: preliminary study adding emission lines to SEDs using prescription in Jouvel++11.



Machine learning corrections, including emission lines. **Left**: relative PSF size residuals for SEDs with emission lines, when training catalog does not include emission lines. **Right**: residual PSF size residual when both training and testing catalog SEDs contain emission lines.

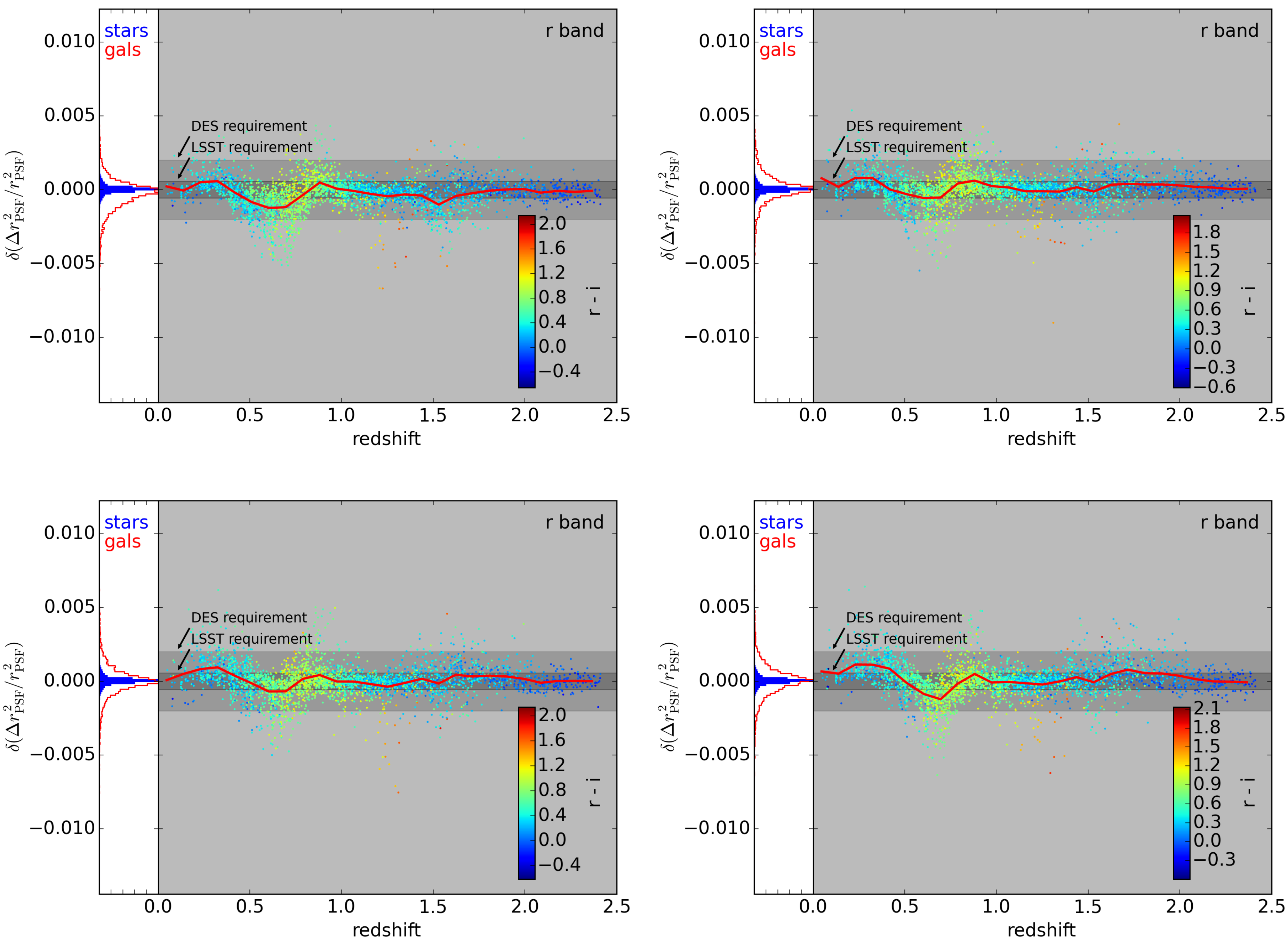
- Machine learning performs similarly with or without emission lines, so long as testing and training catalogs are consistent.

- When catalogs are mismatched, machine learning residuals are ~2 times larger.

Zeropoint systematics

- Chromatic bias corrections derived from photometry may be sensitive to systematic uncertainties in zeropoints.

- To simulate zeropoint uncertainty, we randomly perturb magnitudes in the machine learning testing catalog, but not the training catalog.



PSF size residuals. In each panel normally distributed zeropoint offsets with standard deviation 0.02 mag are applied to each filter.

- Zeropoint uncertainty of 0.02 mag in each band increased PSF size residuals by factor of ~2 in worst case, warranting further study.

How to study galaxies with realistic color gradients?

- Existing studies of chromatic PSF effects on galaxies with color gradients assume simple bulge+disk models.

- To investigate realistic color gradients, Semboloni++13 suggested the using multiband HST imaging as input:

HST image model

$$\underbrace{I_i(\vec{x})}_{\text{images}} = \int \underbrace{T_i(\lambda)}_{\text{filters}} \left[\underbrace{\Pi(\vec{x},\lambda)}_{\text{PSF}} * \underbrace{f(\vec{x},\lambda)}_{\text{galaxy}}\right] \mathrm{d}\lambda + \underbrace{\eta_i(\vec{x})}_{\text{noise}}$$

Goal: determine $f(\vec{x},\lambda)$ from $I_i(\vec{x})$, $T_i(\lambda)$, and $\Pi(\vec{x},\lambda)$

Make problem tractable by modeling galaxy as sum over products of (asserted) SEDs and achromatic profiles:

$$f(\vec{x},\lambda) = \sum_j \underbrace{S_j(\lambda)}_{\text{SED}} \underbrace{a_j(\vec{x})}_{\text{spatial component}}$$

Fourier transforming makes the problem linear:

$$\begin{aligned}
\tilde{I}_i(\vec{k}) &= \int T_i(\lambda) \sum_j S_j(\lambda) \tilde{\Pi}(\vec{k},\lambda) \tilde{a}_j(\vec{k})\, \mathrm{d}\lambda + \tilde{\eta}_i(\vec{k}) \\
&= \sum_j \left[\int T_i(\lambda) S_j(\lambda) \tilde{\Pi}(\vec{k},\lambda)\, \mathrm{d}\lambda\right] \tilde{a}_j(\vec{k}) + \tilde{\eta}_i(\vec{k}) \\
&= \sum_j \tilde{\Pi}^{\mathrm{eff}}_{ij}(\vec{k}) \tilde{a}_j(\vec{k}) + \tilde{\eta}_i(\vec{k})
\end{aligned}$$

Define effective PSF for jth SED and ith filter:

$$\tilde{\Pi}^{\mathrm{eff}}_{ij}(\vec{k}) = \int T_i(\lambda) S_j(\lambda) \tilde{\Pi}(\vec{k},\lambda)\, \mathrm{d}\lambda$$

Solving the system

Assume input images have stationary noise correlation function…

$$\xi_i(\vec{x}_l - \vec{x}_m) = \xi_i(\Delta\vec{x}) = \langle \eta_i(\vec{x}_l)\eta_i(\vec{x}_m) \rangle$$

… and power spectrum:

$$P_i(\vec{k}) = \int \xi_i(\Delta\vec{x}) e^{-2\pi i \vec{k}\cdot\Delta\vec{x}}\, \mathrm{d}(\Delta\vec{x})$$

which gives variance of each Fourier mode:

$$\langle \tilde{\eta}_i^*(\vec{k_l})\tilde{\eta}_i(\vec{k_m}) \rangle = \delta(\vec{k_l} - \vec{k_m}) P_i(\vec{k_l})$$

Likelihood for $\tilde{a}_j(\vec{k})$ is then

$$-2\log\mathcal{L}(\vec{k}) = \sum_i \frac{1}{P_i(\vec{k})} \left| \tilde{I}_i(\vec{k}) - \sum_j \tilde{\Pi}^{\mathrm{eff}}_{ij}(\vec{k})\tilde{a}_j(\vec{k}) \right|^2$$

This is an independent least squares problem for each Fourier mode $\vec{k}$

with solution (omitting matrix subscripts)

$$\tilde{a} = \left(\tilde{\Pi}^{\mathrm{eff},\dagger} W \tilde{\Pi}^{\mathrm{eff}}\right)^{-1} \tilde{\Pi}^{\mathrm{eff},\dagger} W \tilde{I}$$

where weight matrix W is

$$W_{ij} = \delta_{ij}/\sqrt{P_i}$$

The covariance matrix for $\tilde{a}$ is given by $\Sigma = \left(\tilde{\Pi}^{\mathrm{eff},\dagger} W \tilde{\Pi}^{\mathrm{eff}}\right)^{-1}$

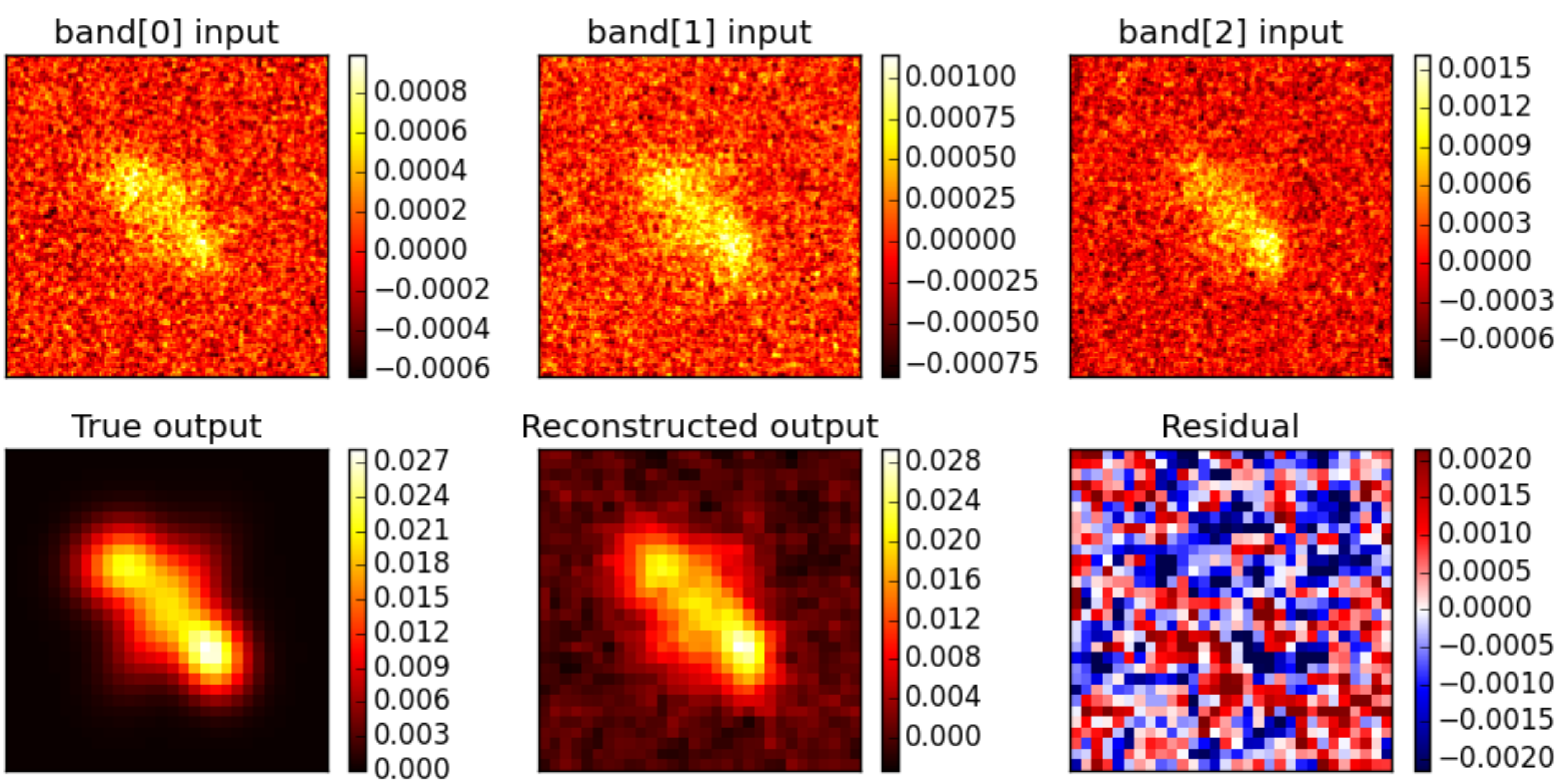
Simulated example

We can now compute what the galaxy would look like under a different (chromatic) PSF and through a different bandpass:

$$\tilde{I}^{\text{out}}(\vec{k}) = \int T^{\text{out}}(\lambda)\tilde{\Pi}^{\text{out}}(\vec{k},\lambda)\sum_j S_j(\lambda)\tilde{a}_j(\vec{k})\,\mathrm{d}\lambda$$

$$= \sum_j \tilde{\Pi}_j^{\text{out,eff}}(\vec{k})\tilde{a}_j(\vec{k})$$

Example projecting simulated HST images into the Euclid PSF and filter.



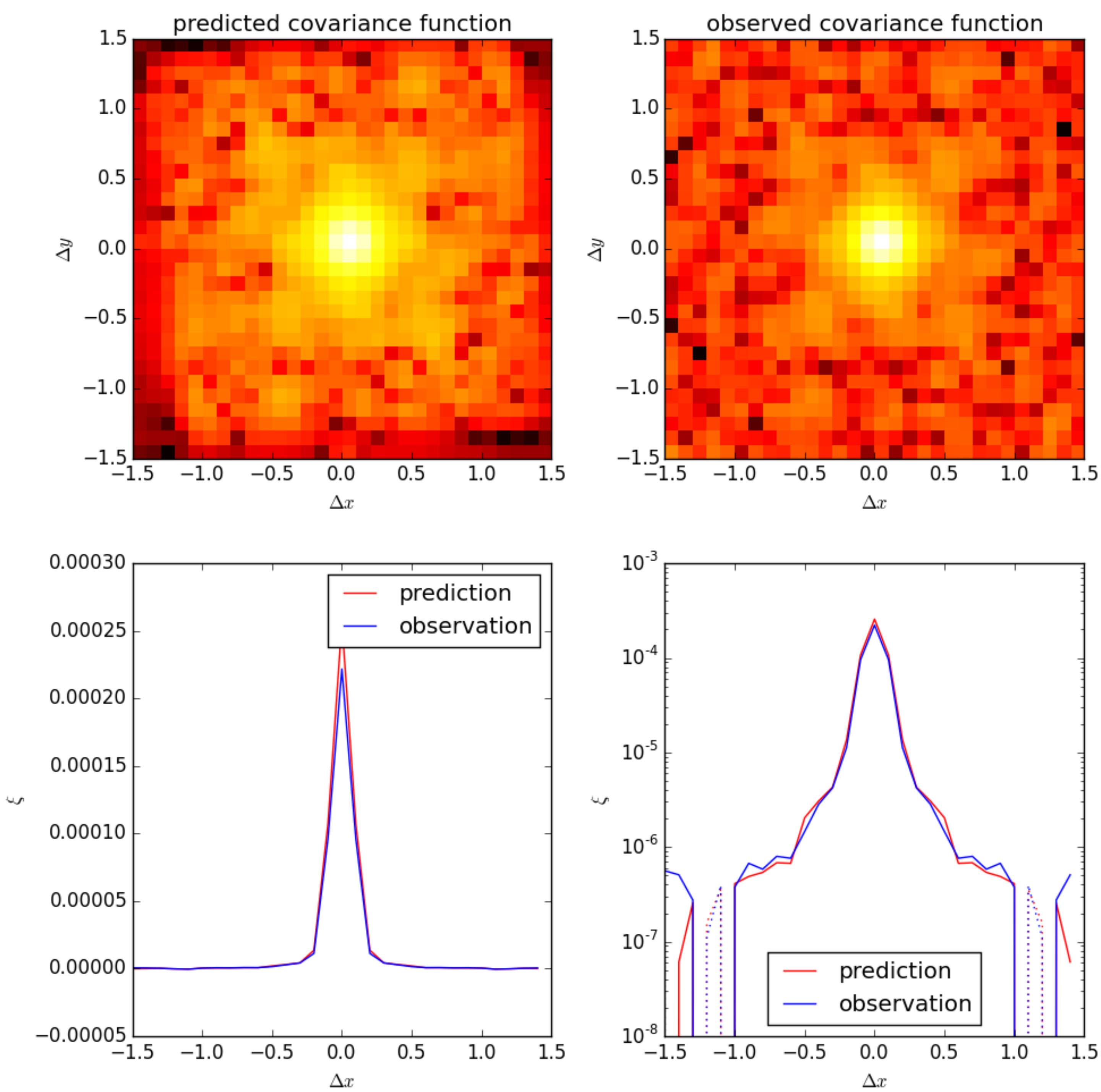
Top row: simulated noisy HST-like images in three bands of galaxy with color gradients. **Bottom left:** noise-free image of same galaxy, but convolved with Euclid-like chromatic PSF and integrated over Euclid-like filter. **Bottom middle:** output reconstructed using multi-band HST-like images on top row as input instead of true model. **Bottom right:** residuals.

Noise propagation

The output noise power spectrum follows from standard linear propagation of errors:

$$P^{\mathrm{out}}(\vec{k}) = \sum_{jj'} \tilde{\Pi}_j^{\mathrm{out,eff},*}(\vec{k})\Sigma_{jj'}(\vec{k})\tilde{\Pi}_{j'}^{\mathrm{out,eff}}(\vec{k}).$$

To investigate the algorithm's ability to accurately propagate noise, we generated 1000 pairs of input images (in 2 simulated filters) of pure noise, and then measured the noise correlations in the output images.



Top left: predicted 2D noise correlation function. **Top right:** measured correlation function. **Bottom left:** 1D slice through correlation functions on linear scale. **Bottom right:** 1D slice through correlation functions on log scale.

Match is reasonable, except at origin (i.e., except overall variance) where prediction is ~10% too high.